|luP

NASA TECHNICAL TRANSLATION NASA TT F-14,777

DIFFUSION OF RADIATION IN A MEDIUM LIMITED BY A GLASSY SURFACE

S. D. Gutshabash

Translation of: "O diffuzii izlucheniya v srede ogranichennoy zerkal'no otrazhayushchey poverkhnost'yu," Optika okean i atmosfery (Optics of the Ocean and the Atmosphere), Edited by K. S. Shifrin, Leningrad, "Nauka" Press, 1972, pp 56-62.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, DC 20546 SEPTEMBER 1973

DIFFUSION OF RADIATION IN A MEDIUM LIMITED BY A GLASSY SURFACE

S. D. Gutshabash

The exact solution for the problem of diffusion of radiation in a semi-infinite medium adjacent to a glassy surface in the case of isotropic scattering is well known [1]. The unknown brightness coefficient $\bar{\rho}(\eta, \zeta)$, which depends on the two variables η and ζ, may be expressed in terms of certain auxiliary functions, each of which depends on one of the variables η or ζ. /56*

It will be shown below that $\bar{\rho}(\eta, \zeta)$ cannot be represented uniquely by means of auxiliary functions, and in this connection the auxiliary functions are determined by different equations. /57

Let us assume that radiation enters an isotropically scattering medium of a semi-infinite optical thickness, limited by a glassy surface. The flux of this radiation through a unit surface at the medium boundary equals πS, and the angle of incidence $i = \arccos \zeta$. If the boundary is not taken into account, then the strength of the radiation $I(\eta, \zeta)$, which is diffusely reflected at the angle $\vartheta = \arccos \eta$, is determined by the formula

$$I(\eta, \zeta) = S\rho(\eta, \zeta)\zeta, \tag{1}$$

*Numbers in the margin indicate pagination of original foreign text.

where

$$\rho(\eta, \zeta) = \frac{\lambda}{4} \frac{\varphi(\eta)\varphi(\zeta)}{\eta+\zeta}; \tag{2}$$

where λ is the quantum survival probability: $\varphi(\eta) = 1 + 2\eta \int_0^1 \rho(\eta, \zeta) d\zeta$ — Ambartsumyan function. In the presence of a glassy surface, for the strength of diffusely reflected radiation

$$\bar{I}(\eta, \zeta) = S\bar{\rho}(\eta, \zeta)\zeta, \tag{3}$$

we have the equation

$$\bar{I}(\eta, \zeta) = I(\eta, \zeta) + 2\pi \int_0^1 \frac{I(\eta', \zeta)}{\pi} p(\eta') \bar{\rho}(\eta, \eta') \eta' d\eta', \tag{4}$$

in which $p(\eta)$ is the portion of radiation reflected from a limiting surface. Substituting Expressions (1) — (3) in (4), we obtain an integral equation which is satisfied by the desired brightness coefficient $\bar{\rho}(\eta, \zeta)$

$$\bar{\rho}(\eta, \zeta) = \rho(\eta, \zeta) + 2 \int_0^1 \rho(\eta', \zeta) p(\eta') \bar{\rho}(\eta, \eta') \eta' d\eta'. \tag{5}$$

This equation was obtained previously in a different way. In order to separate the variables in Equation (5), we set

$$\bar{\rho}(\eta, \zeta) = \frac{\lambda}{4} \left[\frac{A(\eta, \zeta)}{\eta+\zeta} + \frac{B(\eta, \zeta)}{\eta+\zeta} \right]. \tag{6}$$

Substituting (6) in (5) and equating the coefficients when /58
$1/(\eta + \zeta)$ and $1/(\eta - \zeta)$ in the left and right sides of the equation obtained, we obtain the system of equations for $A(\eta, \zeta)$ and $B(\eta, \zeta)$

$$A(\eta, \zeta) = \varphi(\eta)\varphi(\zeta) + \frac{\lambda}{2} \int_0^1 \varphi(\eta')\varphi(\zeta) p(\eta') \times \left(\frac{\eta}{\eta-\eta'} - \frac{\zeta}{\eta'+\zeta} \right) B(\eta, \eta') d\eta'; \tag{7}$$

$$B(\eta, \zeta)=\frac{\lambda}{2}\int_0^1 \varphi(\eta')\varphi(\zeta)p(\eta')\left(\frac{\eta}{\eta+\eta'}-\frac{\zeta}{\eta+\zeta}\right)A(\eta, \eta')\,d\eta'. \tag{8}$$

Based on the principle of reversibility for optical phenomena, we may assume that $\bar{\rho}(\eta, \zeta)$ must be a symmetrical function of η and ζ. Consequently, $A(\eta, \zeta)$ must be a symmetrical function, and $B(\eta, \zeta)$ must be an asymmetrical function with respect to η and ζ

$$A(\eta, \zeta)=A(\zeta, \eta); \quad B(\eta, \zeta)=-B(\zeta, \eta). \tag{9}$$

Therefore, we may set

$$\begin{cases} A(\eta, \zeta)=\alpha(\eta)\alpha(\zeta)-\beta(\eta)\beta(\zeta); \\ B(\eta, \zeta)=\alpha(\eta)\beta(\zeta)-\alpha(\zeta)\beta(\eta). \end{cases} \tag{10}$$

Substituting (10) in Equations (7) and (8), we obtain

$$\begin{aligned} \alpha(\eta)&=\varphi(\eta)+2\eta\int_0^1 \rho(\eta, \eta')p(\eta')\beta(\eta')\,d\eta'; \\ \beta(\eta)&=2\eta\int_0^1 \rho(\eta, \eta')p(\eta')\alpha(\eta')\,d\eta'. \end{aligned} \tag{11}$$

The functions $\alpha(\eta)$ and $\beta(\eta)$ must satisfy the equations

$$\begin{aligned} \alpha(\eta)&=\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1 \frac{\varphi(\eta)p(\eta')}{\eta-\eta'}B(\eta, \eta')\,d\eta'; \\ \beta(\eta)&=-\frac{\lambda}{2}\eta\int_0^1 \frac{\varphi(\eta')p(\eta')}{\eta+\eta'}A(\eta, \eta')\,d\eta'. \end{aligned} \tag{12}$$

It may be shown that if $\alpha(\eta)$ and $\beta(\eta)$ satisfy Equations (11), then they satisfy Equations (12). A similar solution was given in [1]. Let us set /59

$$\begin{aligned} A(\eta, \zeta)&=\alpha(\eta)\beta(\zeta)+\alpha(\zeta)\beta(\eta); \\ B(\eta, \zeta)&=\alpha(\eta)\beta(\zeta)-\alpha(\zeta)\beta(\eta), \end{aligned} \tag{13}$$

Then $\alpha(\eta)$ and $\beta(\eta)$ are determined by the equations

$$\begin{aligned}\beta(\zeta)&=C_1\varphi(\zeta)-2\zeta\int_0^1 \rho(\eta',\zeta)p(\eta')\beta(\eta')\,d\eta';\\ \alpha(\zeta)&=C_2\varphi(\zeta)+2\zeta\int_0^1 \rho(\eta',\zeta)p(\eta')\alpha(\eta')\,d\eta',\end{aligned}\tag{14}$$

and in addition

$$\begin{aligned}\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1\frac{B(\eta,\eta')}{\eta-\eta'}\varphi(\eta')p(\eta')\,d\eta'&=C_1\alpha(\eta)+C_2\beta(\eta);\\ \frac{\lambda}{2}\eta\int_0^1\frac{A(\eta,\eta')}{\eta+\eta'}\varphi(\eta')p(\eta')\,d\eta'&=C_1\alpha(\eta)-C_2\beta(\eta).\end{aligned}\tag{15}$$

Thus, the constants C_1 and C_2 satisfy the condition

$$2C_1C_2=1.\tag{16}$$

We shall show that, if the functions $\alpha(\eta)$ and $\beta(\eta)$ satisfy Equations (14), then they satisfy Equation (15). Actually, using Expressions (3) and (13), we obtain

$$\begin{aligned}\frac{\lambda}{2}\eta\int_0^1\frac{A(\eta,\eta')}{\eta+\eta'}\varphi(\eta')p(\eta')\,d\eta'=\frac{\alpha(\eta)}{\varphi(\eta)}2\eta\int_0^1\rho(\eta,\eta')\beta(\eta')p(\eta')\,d\eta'+\\ +\frac{\beta(\eta)}{\varphi(\eta)}2\eta\int_0^1\rho(\eta,\eta')\alpha(\eta')p(\eta')\,d\eta';\end{aligned}\tag{17}$$

and by means of Expression (14), we obtain

$$\begin{aligned}\frac{\lambda}{2}\eta\int_0^1\frac{A(\eta,\eta')}{\eta+\eta'}\varphi(\eta')p(\eta')\,d\eta'=\frac{\alpha(\eta)}{\varphi(\eta)}[C_1\varphi(\eta)-\beta(\eta)]+\\ +\frac{\beta(\eta)}{\varphi(\eta)}[\alpha(\eta)-C_2\varphi(\eta)]=C_1\alpha(\eta)-C_2\beta(\eta).\end{aligned}\tag{18}$$

Thus, the second of the equations (15) will be satisfied. Let /60
us show that the first of these equations is satisfied. Taking into account (13) and (14), we shall have

$$
\begin{aligned}
&\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1\frac{B(\eta,\eta')}{\eta-\eta'}\varphi(\eta')p(\eta')\,d\eta'=\varphi(\eta)+\Big[C_2\varphi(\eta)+\\
&+2\eta\int_0^1\rho(\eta,\eta')p(\eta')\alpha(\eta')\,d\eta'\Big]\frac{\lambda}{2}\eta\int_0^1\frac{\beta(\eta')}{\eta-\eta'}p(\eta')\varphi(\eta')\,d\eta'-\\
&-\Big[C_1\varphi(\eta)-2\eta\int_0^1\rho(\eta,\eta')p(\eta')\beta(\eta')\,d\eta'\Big]\times\\
&\times\frac{\lambda}{2}\eta\int_0^1\frac{\alpha(\eta')}{\eta-\eta'}p(\eta')\varphi(\eta')\,d\eta';
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1\frac{B(\eta,\eta')}{\eta-\eta'}\varphi(\eta')p(\eta')\,d\eta'=\varphi(\eta)+\\
&+C_2\varphi(\eta)\frac{\lambda}{2}\eta\int_0^1\frac{\beta(\eta')}{\eta-\eta'}p(\eta')\varphi(\eta')\,d\eta'-C_1\varphi(\eta)\frac{\lambda}{2}\eta\int_0^1\frac{\alpha(\eta')}{\eta-\eta'}\times\\
&\times p(\eta')\varphi(\eta')\,d\eta'+\frac{\lambda^2}{4}\eta^2\int_0^1\int_0^1\frac{\varphi(\eta)\varphi(\eta')}{\eta+\eta'}p(\eta')\alpha(\eta')\frac{\beta(\eta'')}{\eta-\eta''}\times\\
&\times p(\eta'')\varphi(\eta'')\,d\eta'\,d\eta''+\frac{\lambda^2}{4}\eta^2\int_0^1\int_0^1\frac{\varphi(\eta)\varphi(\eta')}{\eta+\eta'}p(\eta')\beta(\eta')\times\\
&\times\frac{\alpha(\eta'')}{\eta-\eta''}p(\eta'')\varphi(\eta'')\,d\eta'\,d\eta''.
\end{aligned}
\tag{20}
$$

Since

$$
\frac{\eta}{(\eta+\eta')(\eta-\eta'')}=\left(\frac{\eta'}{\eta+\eta'}+\frac{\eta''}{\eta-\eta''}\right)\frac{1}{\eta'+\eta''},
\tag{21}
$$

then the Expression (20) obtained above may be rewritten in the form

$$
\begin{aligned}
&\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1\frac{B(\eta,\eta')}{\eta-\eta'}\varphi(\eta')p(\eta')\,d\eta'=\varphi(\eta)+C_1 2\eta\times\\
&\times\int_0^1\rho(\eta,\eta')p(\eta')\alpha(\eta')\,d\eta'-C_2 2\eta\int_0^1\rho(\eta,\eta')\beta(\eta')p(\eta')\,d\eta'.
\end{aligned}
\tag{22}
$$

Using Equation (14) again, we obtain /61

$$\varphi(\eta)+\frac{\lambda}{2}\eta\int_0^1\frac{B(\eta,\eta')}{\eta-\eta'}\varphi(\eta')p(\eta')d\eta'=\varphi(\eta)-2C_1C_2\varphi(\eta)+ +C_1\alpha(\eta)+C_2\beta(\eta). \tag{23}$$

Thus, the first of the equations (15) is satisfied if the constants C_1 and C_2 satisfy (16).

Thus, the solution of Equation (2) may be written in the form (6), where $A(\eta, \zeta)$ and $B(\eta, \zeta)$ are determined by Expressions (13), and the auxiliary functions $\alpha(\eta)$ and $\beta(\eta)$ form a single parametric family of functions (14) which depend on the parameter C_1 (or C_2), and C_1 and C_2 are related with Condition (16).

It may be readily seen that the desired brightness coefficent $\bar{\rho}(\eta, \zeta)$ does not depend on the value of the parameter C_1. Utilizing Expression (16), the equations for the auxiliary functions (14) may be written in the form

$$\beta(\zeta)=C_1\varphi(\zeta)-2\zeta\int_0^1\rho(\eta',\zeta)p(\eta')\beta(\eta')d\eta';$$
$$\alpha(\zeta)=\frac{\lambda}{2C_1}\varphi(\zeta)+2\zeta\int_0^1\rho(\eta',\zeta)p(\eta')\alpha(\eta')d\eta'. \tag{24}$$

If $\Gamma_1(\eta, \zeta)$ is the resolvent of the first of these equations, and $\Gamma_2(\eta, \zeta)$ is the resolvent of the second equation, then the solutions of these equations may be written in the form

$$\beta(\zeta)=C_1\left[\varphi(\zeta)-\int_0^1\Gamma_1(\eta',\zeta)\varphi(\eta')d\eta'\right];$$
$$\alpha(\zeta)=\frac{1}{2C_1}\left[\varphi(\zeta)+\int_0^1\Gamma_2(\eta',\zeta)\varphi(\eta')d\eta'\right]. \tag{25}$$

Substituting (24) into (13), we find that $A(\eta, \zeta)$ and $B(\eta, \zeta)$, and consequently the desired brightness coefficient $\bar{\rho}(\eta, \zeta)$, do not depend on C_1.

For each specific law governing the reflection of light by a surface, which is characterized by the function $p(\eta)$, the auxiliary functions may be found numerically, and we may set C_1 equal to any number differing from zero.

Thus, if $A(\eta, \zeta)$ and $B(\eta, \zeta)$ are desired in the form (10), then the auxiliary functions are determined by the system of Equations (11). If $A(\eta, \zeta)$ and $B(\eta, \zeta)$ are written in the form (13), then for each of the functions a separate equation is /62
obtained. In numerical calculations, it is more convenient to represent the auxiliary functions in the form (14) (with a valid selection of C_1 and C_2) than in the form (12).

REFERENCES

1. Sobolev, V. V. Perenos izlucheniya v atmosferakh zvezd i planet (Radiation Transport in the Atmospheres of Stars and Planets). Moscow, Gostekhteorizdat, 1956.

Translated for GODDARD SPACE FLIGHT CENTER under contract No. NASw 2483, by SCITRAN, P. O. Box 5456, Santa Barbara, California 93108.

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-14,777		

4. Title and Subtitle	5. Report Date
DIFFUSION OF RADIATION IN A MEDIUM LIMITED BY A GLASSY SURFACE	Sept. 1973
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
S. D. Gutshabash	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
SCITRAN, P. O. Box 5456, Santa Barbara, California 93108	NASw 2483
	13. Type of Report and Period Covered
	Translation
12. Sponsoring Agency Name and Address	14. Sponsoring Agency Code
NASA, Washington, DC 20546	

15. Supplementary Notes

Translation of: O diffuzii izlucheniya v srede ogranichennoy zerkal'no otrazhayushchey poverkhnost'yu

Source: Optika okean i atmosfery (Optics of the Ocean and the Atmosphere), Edited by K. S. Shifrin, Leningrad, "Nauka" Press, 1972, pp 56-62

16. Abstract

Intensity of radiation emerging from the medium limited by the glassy surface is determined by an integral equation. Functions, which are in this equation, depend on several arguments. One can find the solution of these equations depending only on one of these arguments. In the article by using the principle of reversibility for optical phenomena it has been shown that the auxilary functions can be determined from different system of integral equations.

17. Key Words (Selected by Author(s))	18. Distribution Statement

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
		8	